



Recent Spin Results from the PHENIX Detector at RHIC

Astrid Morreale

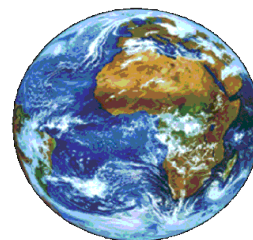
University of California at Riverside

On behalf of the PHENIX collaboration



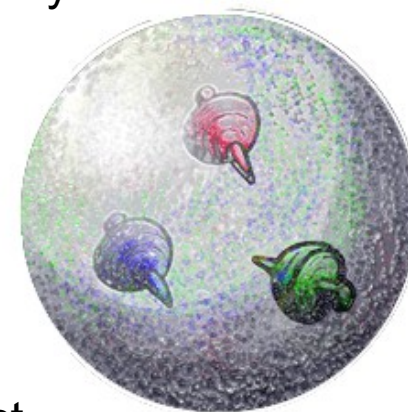
Intrinsic Spin Violates our intuition:

How can an elementary particle such as the e^- be point like and have perpetual angular momentum?



The Proton also has violated our intuition.
The Proton is composed of quarks, gluons and anti quarks.

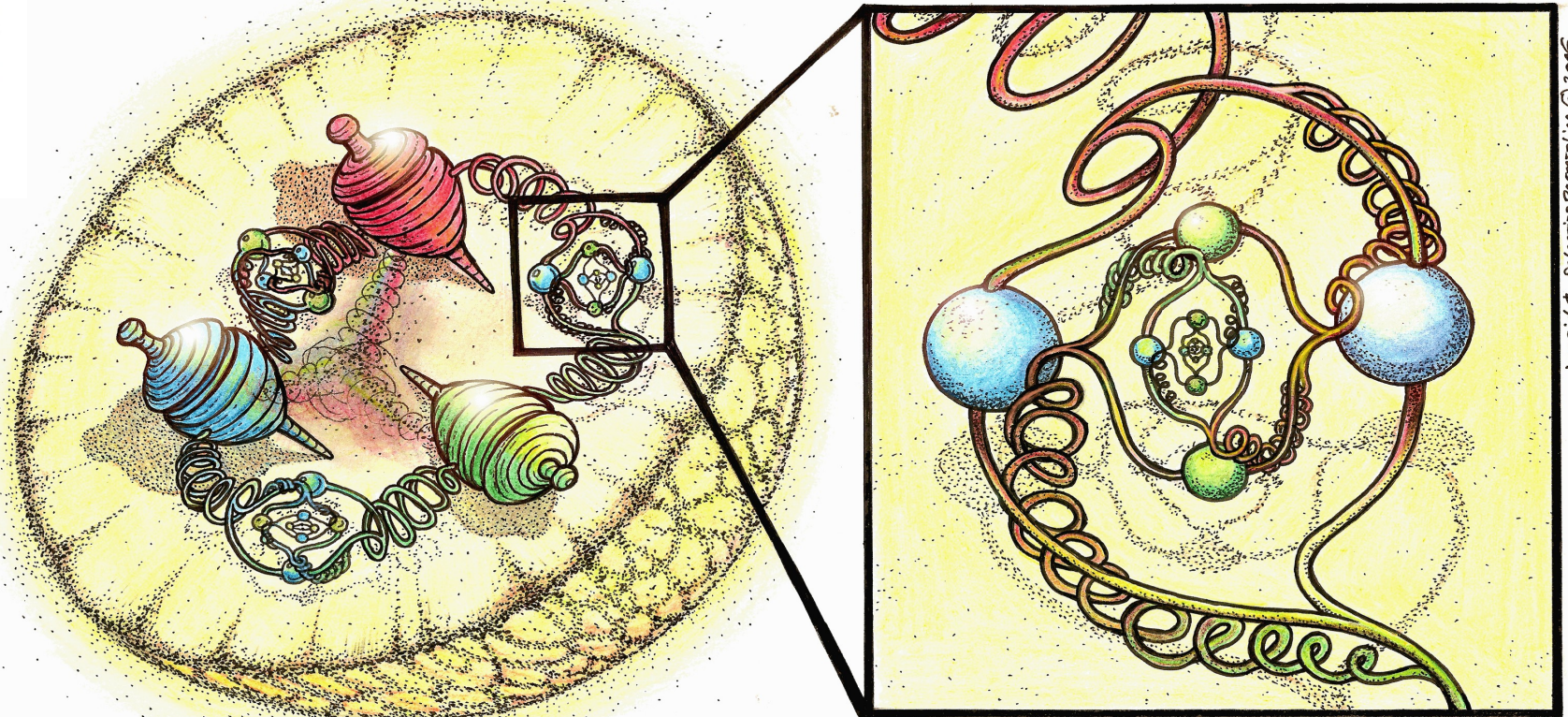
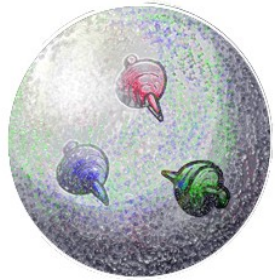
We should expect the proton's spin to be predominately carried by its 3 valence quarks



But as the EMC* Collaboration Found in 1980's
The 3 quarks are only responsible for a small part.
Which means the Proton is a more complicated object.

*Phys. Lett. B206,364 (1988).

The QCD Proton Picture



Astrid & Sébastien Parmetier © 2006

That nucleon has a large anomalous magnetic moment proves that this is not **fundamental** spin1/2 Dirac particle.

Nucleon Spin is Subtle: Quarks, gluons and their angular momentum caused by their high speed motion within the nucleon are contributors to the Nucleon's spin.

Longitudinal Spin Sum Rule:

$$\frac{1}{2} = S_z = \frac{1}{2} \Delta\Sigma + \Delta G + L_z$$

W-production
(pp)

Double Spin
Asymmetries
(pp, SIDIS)

Exclusive
processes
(DVCS, etc)

Transverse Spin Sum Rule??

Bakker, Leader, Trueman
Phys.Rev.D70:114001,2004

Chiral-odd Fragmentation
functions (Collins, IFF, λ)

$$\frac{1}{2} = S_x = \frac{1}{2} \delta\Sigma + L_x$$

Sivers effect??

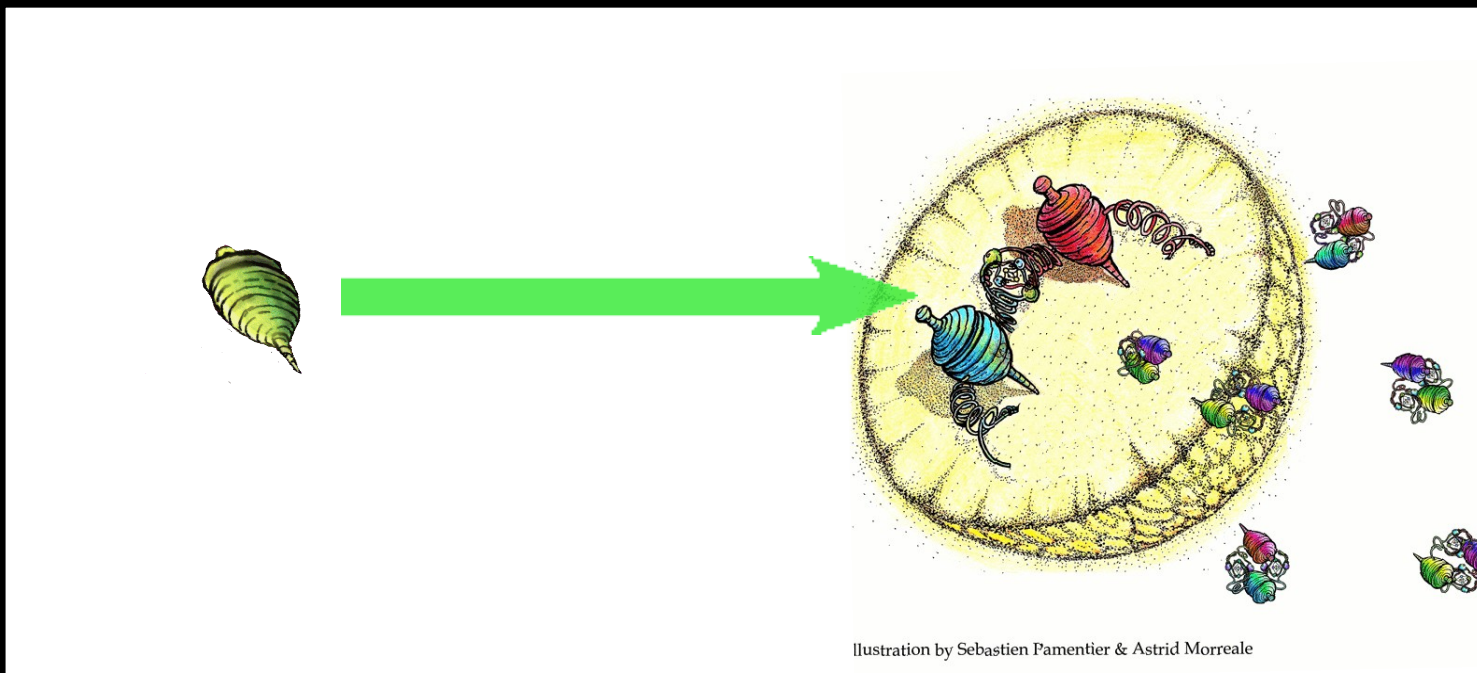
$-\Delta G, \Delta\Sigma$ are the probabilities of finding a parton with spin parallel or anti parallel to the spin of a longitudinally polarized nucleon.

$-L_z$: orbital angular momenta of the quarks and gluons

$-\delta\Sigma$: Difference of quarks with parallel and antiparallel polarization relative to transversely polarized proton

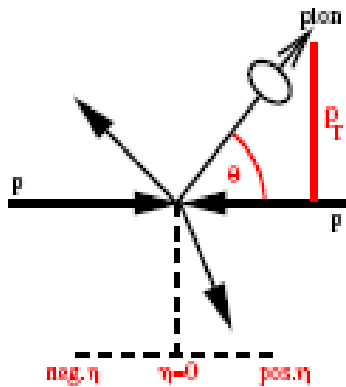


Polarized proton proton Collisions: A QCD LAB

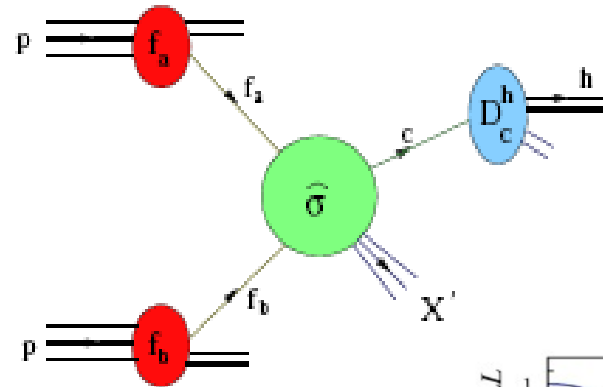


- RHIC provides abundant source of polarized protons and can collide them at high energies.
- Each proton is an ample source of "glue", can be used to probe the gluon's role directly
- RHIC's High Energies keep the interpretation of results clean using pQCD
- We have appropriate detectors to look at such collisions
Phenix is one of these detectors

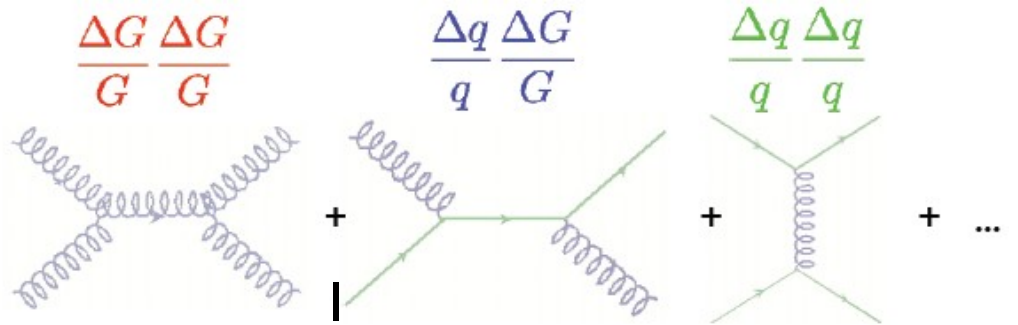
FACTORIZATION → Accessing Δg with Asymmetries



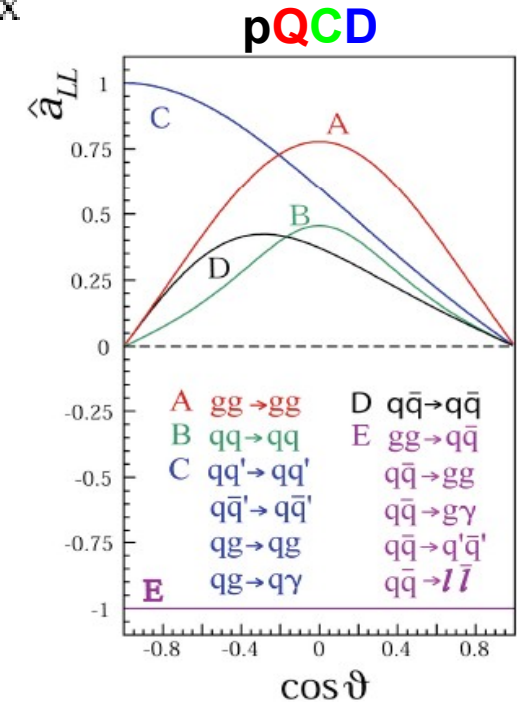
factorization
theorem



$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} = \frac{\sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes d\sigma^{f_a \hat{f}_b \rightarrow fX} \cdot a_{LL}^{f_a \hat{f}_b \rightarrow fX} \otimes D_f^h}{\sum_{a,b} f_a \otimes f_b \otimes d\sigma^{f_a \hat{f}_b \rightarrow fX} \otimes D_f^h}$$



Increasing x , p_T



Hard subprocess asymmetries (LO)

- **Parton distribution Functions (PDF's):** Probability density for finding a particle with a certain longitudinal momentum fraction x at momentum transfer Q^2 . A non-perturbative object, it must be measured!
- **Fragmentation Functions (FF):** The probability for a parton to fragment into a particular hadron carrying a certain fraction of the parton's energy





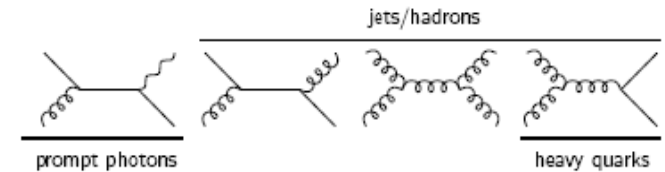
- ❖ For Δg the tools are measurements of helicity cross section asymmetries A_{LL}

$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} = \frac{1}{P_b P_y} \frac{N_{++} - RN_{+-}}{N_{++} + RN_{+-}}$$

(N) Particle Yields
(R) Relative Luminosity
(P) Polarization

Measuring double spin asymmetries in certain final states are the most valuable tool to measure polarized gluon (and quark) distribution functions in the proton.

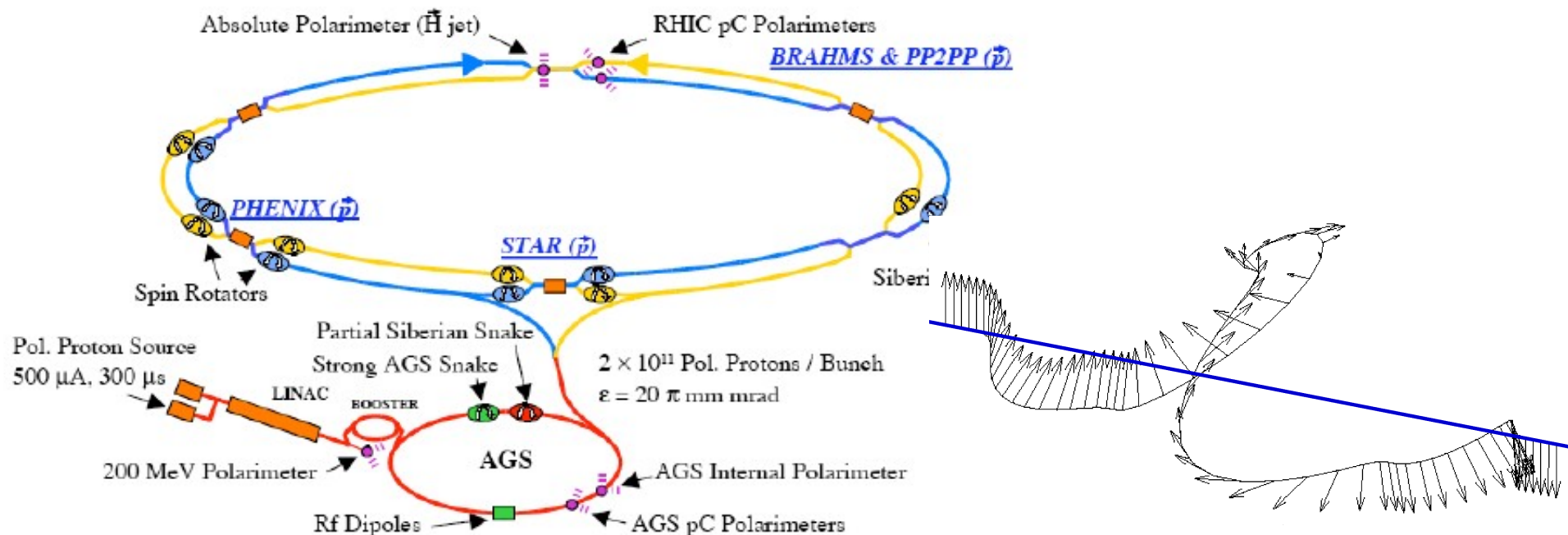
The most accurate way to do so is the study of those processes which can be calculated in the framework of perturbative QCD.



reaction	LO subprocesses	partons probed
$pp \rightarrow \text{jets } X$	$q\bar{q}, qq, qg, gg \rightarrow \text{jet } X$	$\Delta q, \Delta g$
$pp \rightarrow \pi X$	$q\bar{q}, qq, qg, gg \rightarrow \pi X$	$\Delta q, \Delta g$
$pp \rightarrow \gamma X$	$qg \rightarrow q\gamma, q\bar{q} \rightarrow g\gamma$	Δg
$pp \rightarrow Q\bar{Q}X$	$gg \rightarrow Q\bar{Q}, q\bar{q} \rightarrow Q\bar{Q}$	Δg



RHIC Relativistic Heavy Ion Collider: A QCD Laboratory



2 **counter** rotating accelerator storage rings
with collisions at six interaction points

Siberian Snakes: **Depolarizing** resonances
are canceled out by rotating spin by 180
degrees each turn.

Year	[GeV]	Luminosity [pb ⁻¹] (recorded)	Polarization [%]	Figure of Merit P ² L
2003 *	200	0.35	27	25.5nb-1
2004 *	200	0.12	40	19.2nb-1
2005 *	200	3.4	49	816 nb-1
2006 *	200	7.5	55	2268 nb-1
2006 *	62.4	0.08	48	18.4 nb-1

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PNPI, Petersburg Nuclear Physics Institute, Gatchina, Leningrad region, 188300, Russia
 Saint Petersburg State Polytechnical University, St. Petersburg, Russia

14 Countries; 69 Institutions

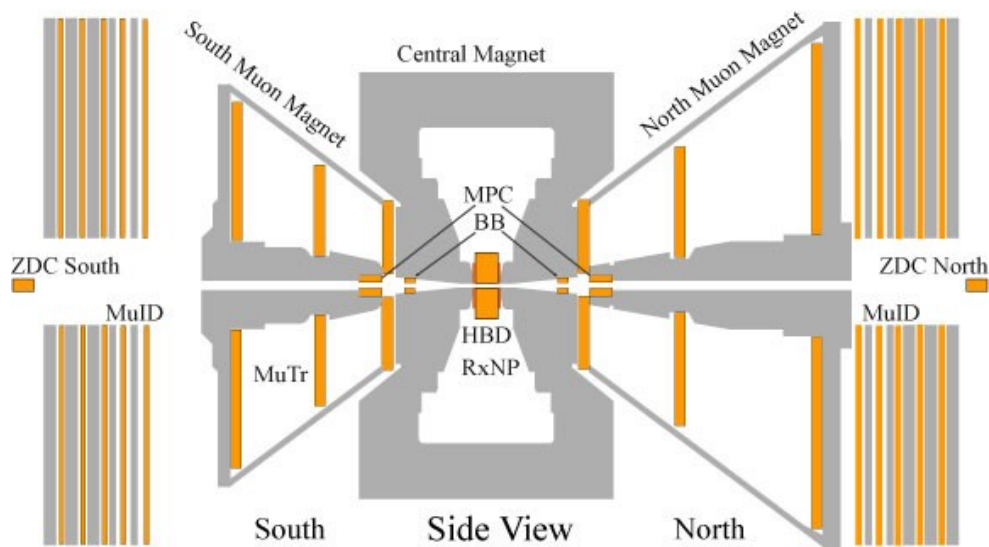
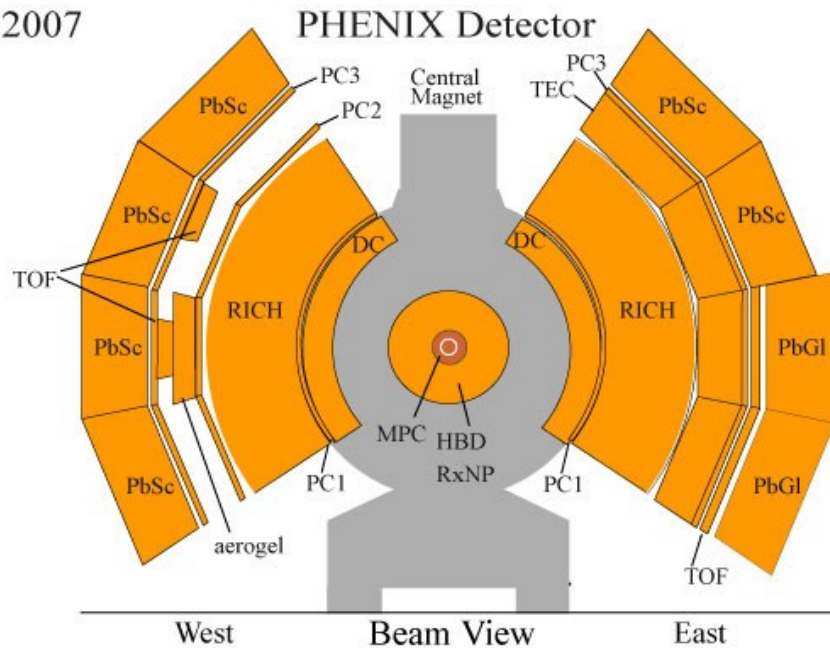


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Visit: <http://www.phenix.bnl.gov/WWW/physics/spin/>

2007



Pseudorapidity

Spatial coordinate describing the angle of a particle relative to the beam axis it is defined as $\eta = -\ln(\tan(\theta/2))$ where theta is the angle relative to the beam axis. η does not depend on the energy of the particle, only on the polar angle of its trajectory

Central Detector *Acceptance:* ($|\eta| < 0.35$, $\phi = 2 \times \pi / 2$):

- $\gamma / \pi^0 / \eta$ detection
 - Electromagnetic Calorimeter: PbSc + PbGl, $\eta < |0.35|$, $\phi = 2 \times 90^\circ$
- π^+ / π^-
 - Drift Chamber
 - Ring Imaging Cherenkov Detector

Muon Arms (forward kinematics ($\sim 1.1 < |\eta| < 2.4$)).

- J/ψ Muon ID/Muon Tracker ($\mu^+ \mu^-$)
- π^0 Electromagnetic Calorimeter (MPC)

Global Detectors:

- Relative Luminosity
 - Beam-Beam Counter (BBC)
 - Zero-Degree Calorimeter (ZDC)
- Local Polarimetry - ZDC

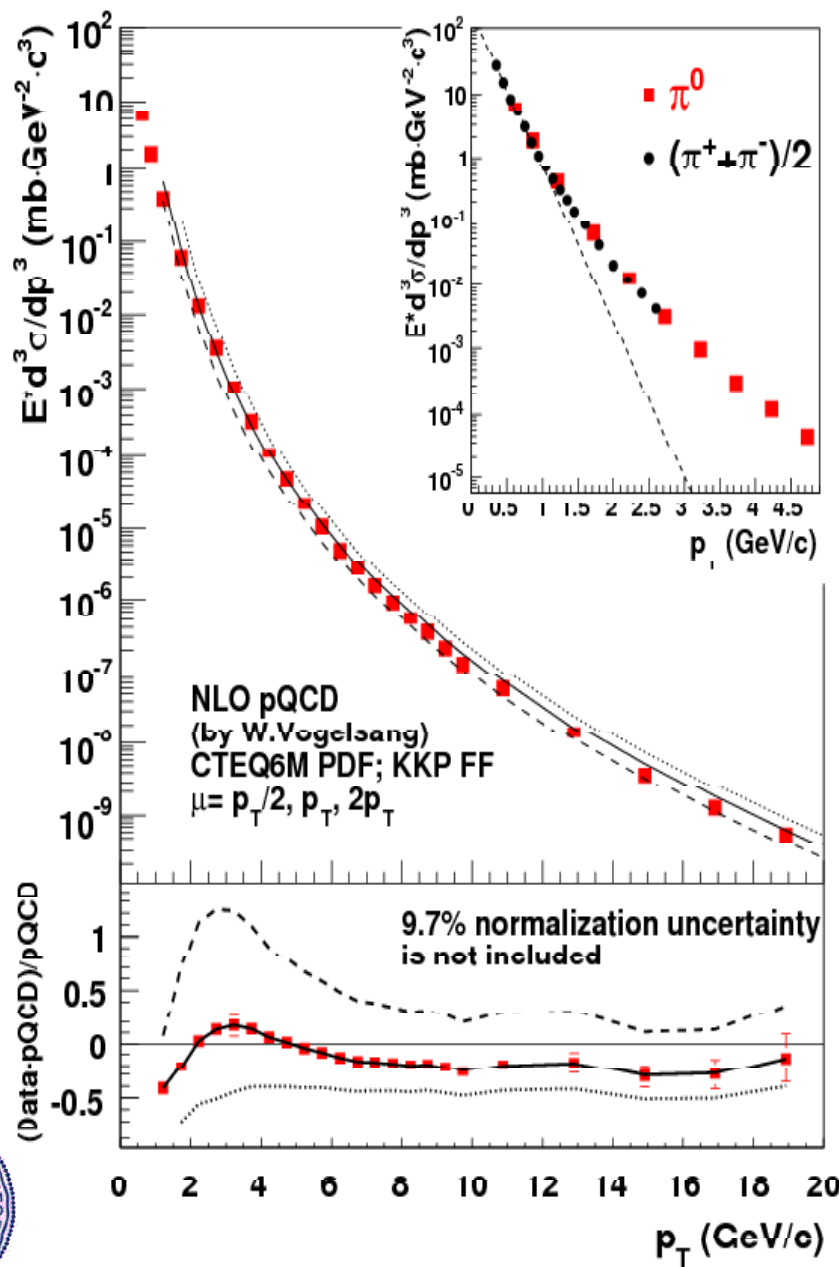


Asymmetries

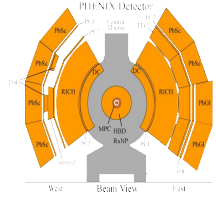
- ❖ Accessing Δg : Inclusive channels $A_{LL}(pp \rightarrow X)$ measurable at Phenix
 - π^0 : wide p_T range, mixture with $gg \rightarrow X$ dominant at low p_T
 - η , similar to π^0 , different FF's.
 - π^\pm , mixture sensitive to $qg \rightarrow qX$ at high p_T
 - Multiparticle clusters (parts of jets), correlated with $\pi^{0,\pm}$
 - Direct photons: p_T range 6-20+ GeV/c, dominated by $qg \rightarrow q\gamma$
 - J/ψ , μ^\pm , e^\pm ($gg \rightarrow cc$)
- ❖ Other Asymmetries measurements at Phenix
 - A_N Left-Right Asymmetries of $\pi^0/\pi^\pm/h^\pm$, J/ψ , forward neutrons
 - D_{LL} Longitudinal spin transfer to Anti- Λ
 - k_T azimuthal asymmetries of hadron Pairs.



PHENIX: π^0 mid-rapidity, 200GeV
Phys. Rev. D 76, 051106 (2007)



- ❖ **Agreement** between data and pQCD theory
- ❖ Shows that pQCD and **unpolarized** PDFs determined in DIS can **describe** pp data
- ❖ Choice of fragmentation function sensitive to gluon fragmentation.
- ❖ Parton distribution Functions (PDF's): Probability density for finding a particle with a certain longitudinal momentum fraction x at momentum transfer Q^2 . A non-perturbative object, it must be measured!

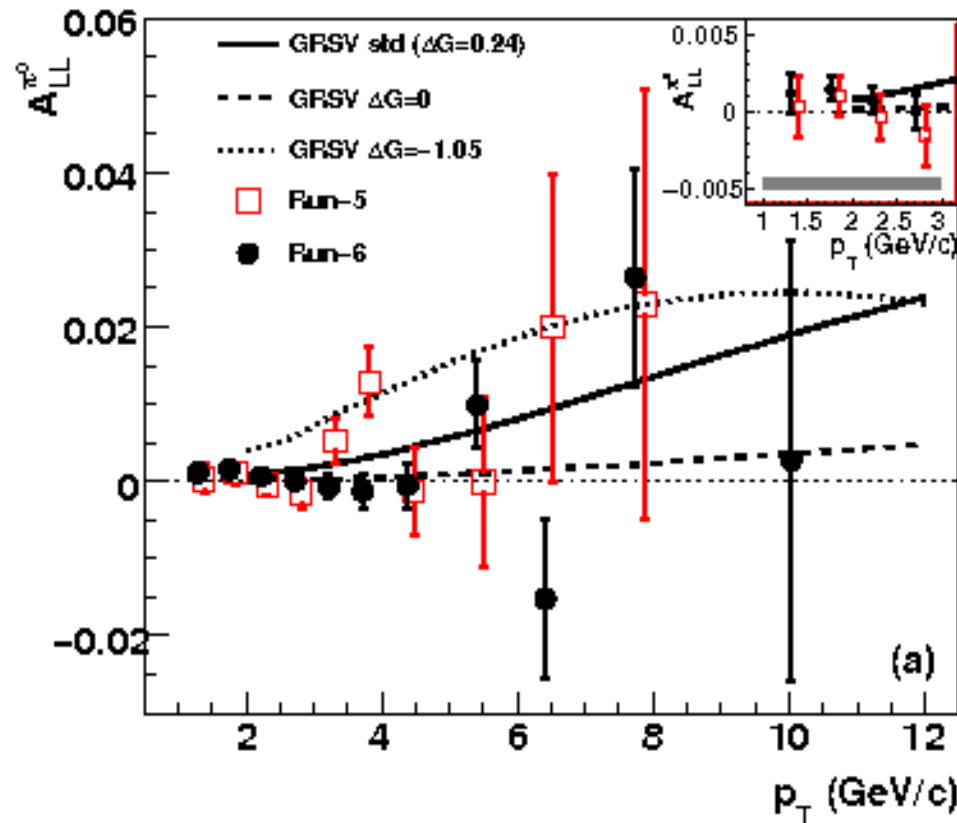


π^0 Asymmetries

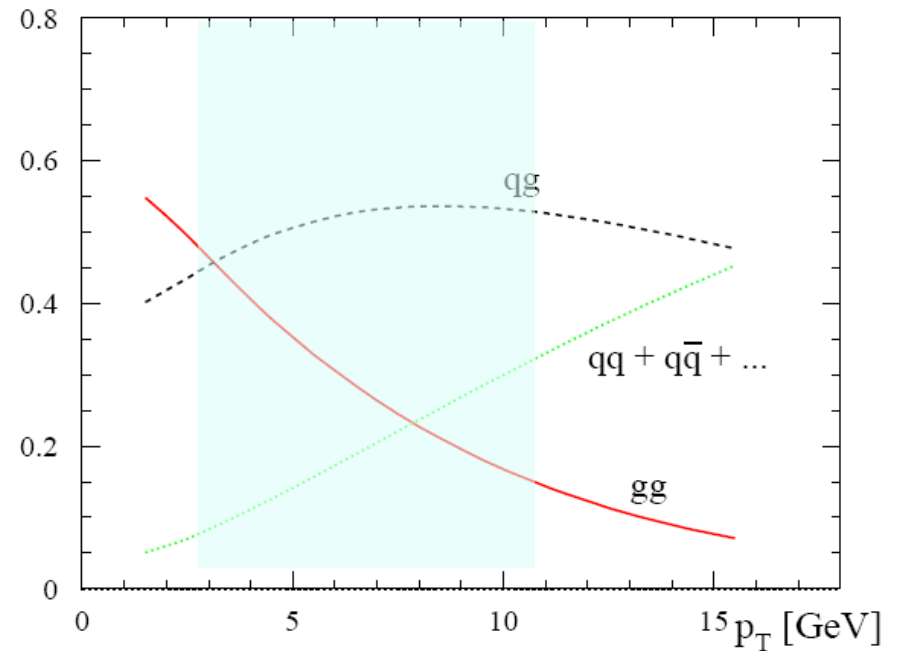
Measured asymmetries for $pp \rightarrow \pi^0 X$ from Run 5, Run 6

Run 3,4,5: PRL 93, 202002; PRD 73, 091102;
Phys. Rev. D 76, 051106 (2007), arXiv:0810.0694

Initial state parton configurations contributing
to unpolarized cross section (Fractions)



Fraction of pion production



W. Vogelsang et al.

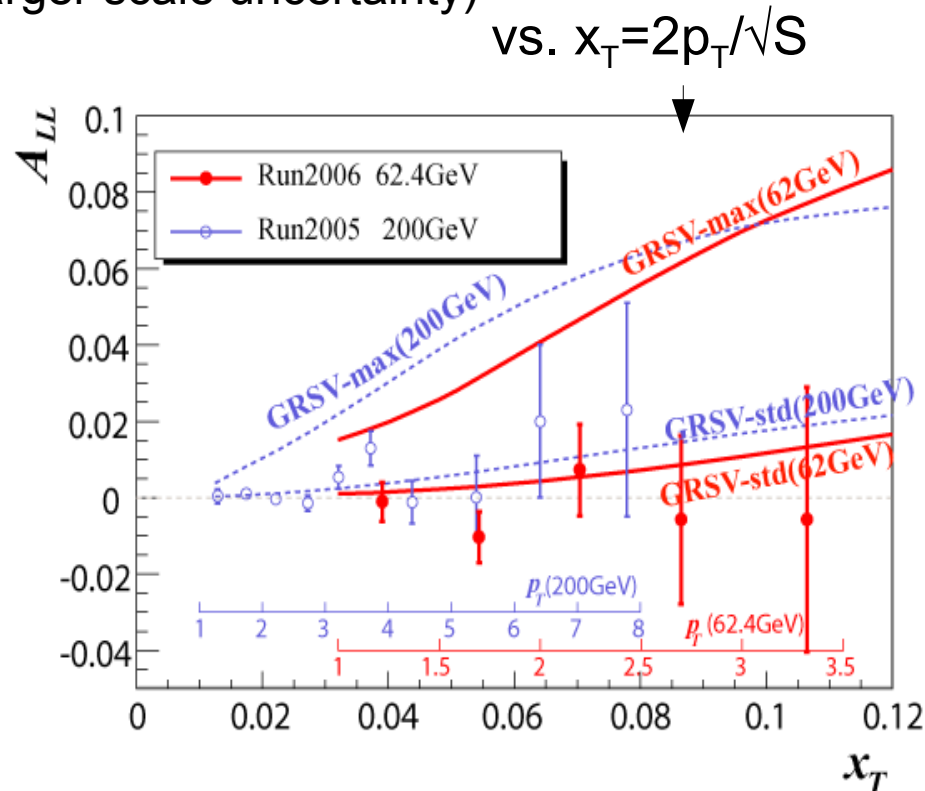
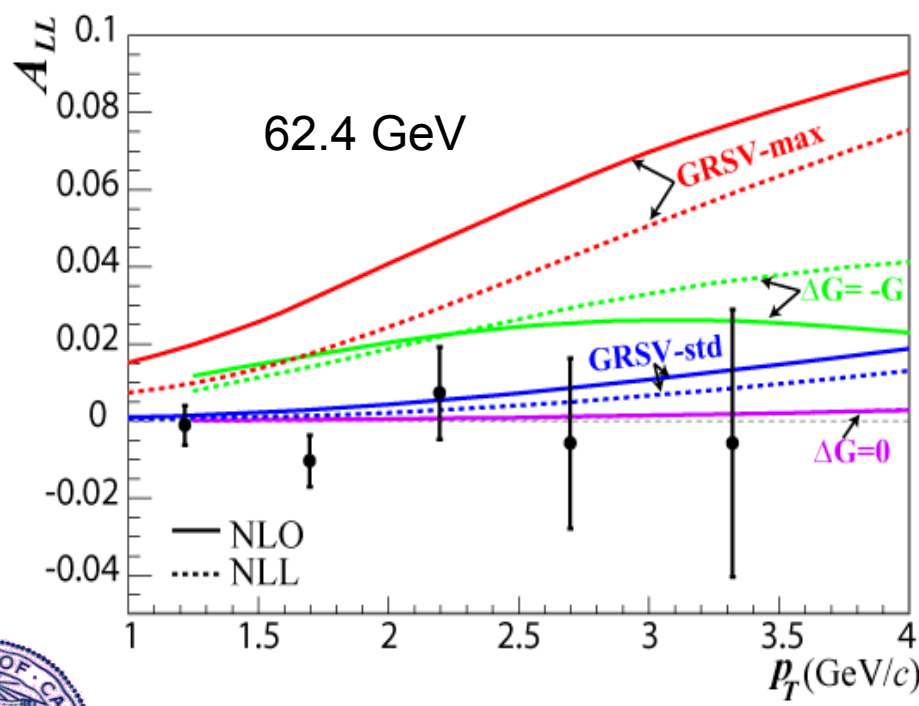
- ❖ Asymmetry of combinatorial background estimated from sidebands and subtracted

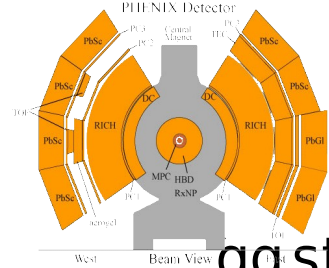
- ❖ Dominated by gg for $p_T < 5$,
- ❖ qg for $p_T > 5$ GeV/c



Information from π^0 Asymmetries

- ❖ Inclusive $\pi^0 A_{LL}$ cannot access $\Delta g(x)$ directly
 - Only sensitive to an average over a wide x range
 - No conclusions about moment of $\Delta g(x)$ possible without a model for its shape
- ❖ More (indirect) information from varying cms energies
 - Higher (500 GeV) \rightarrow lower x
 - Smaller (62 GeV) \rightarrow higher x (and larger scale uncertainty)
 - Phys. Rev. D 79, 012003 (2009)

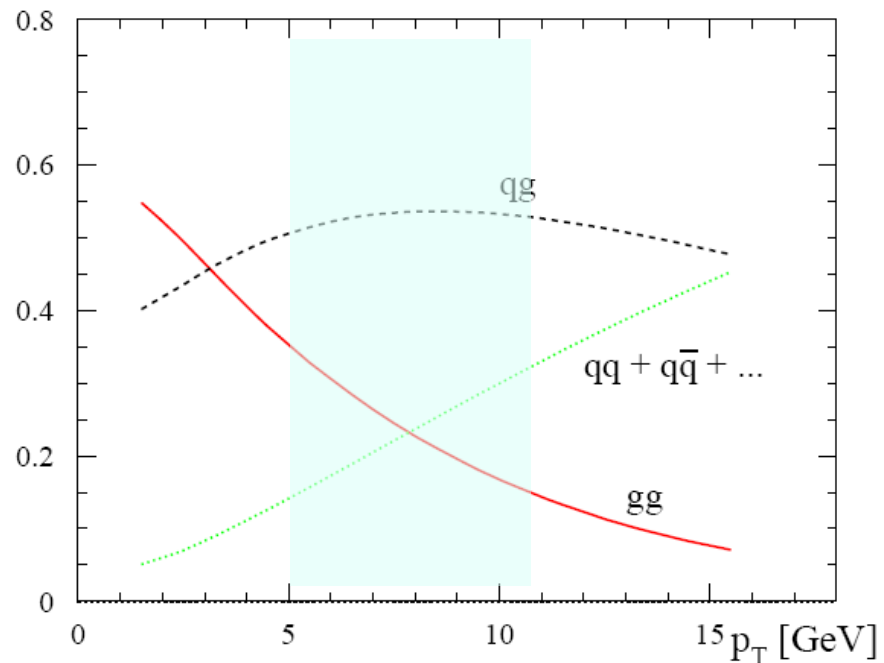
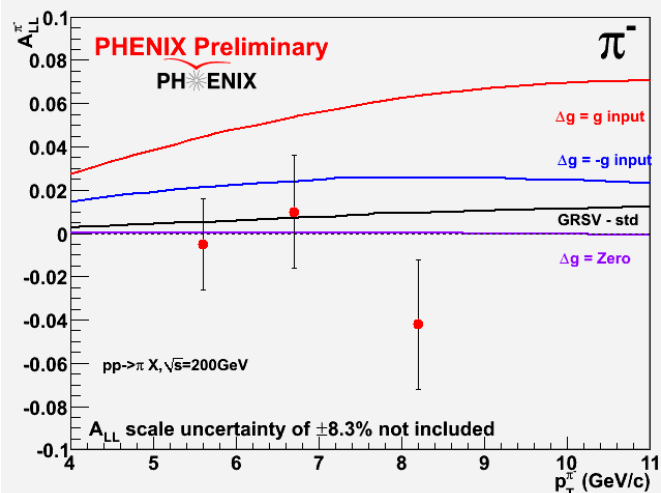
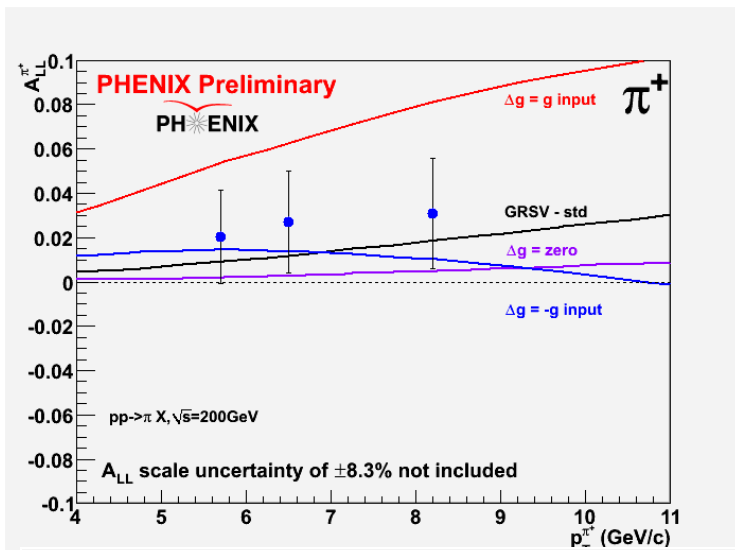




$\pi^{+,-}$ Asymmetries



qg starts to dominate for $p_T > \sim 5 \text{ GeV}$ and $D_u^{\pi^+} > D_u^{\pi^0} > D_u^{\pi^-}$
 Expect sensitivity to sign of ΔG , e.g., positive $A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$



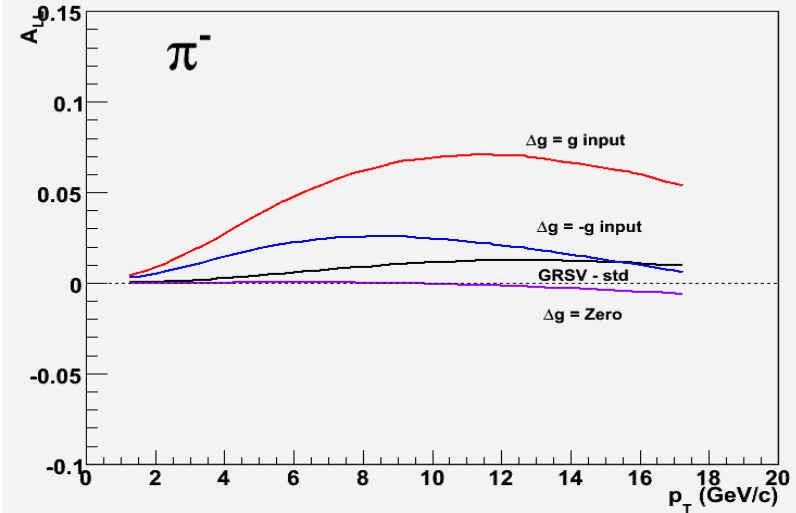
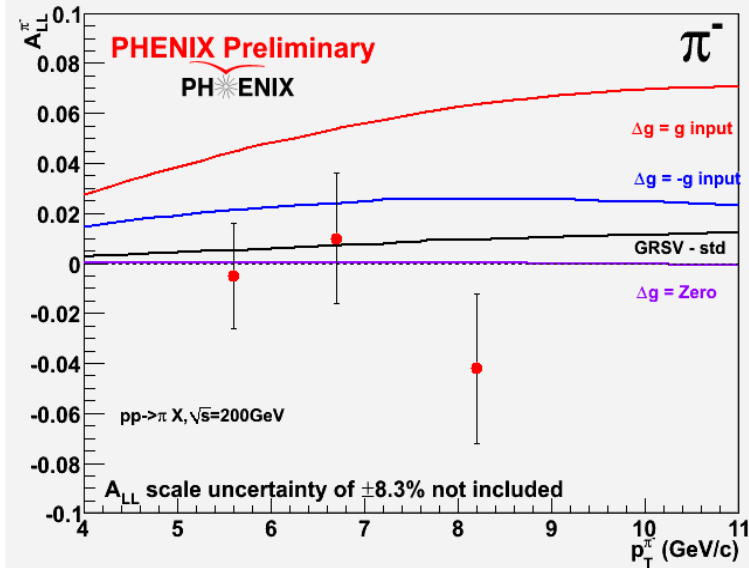
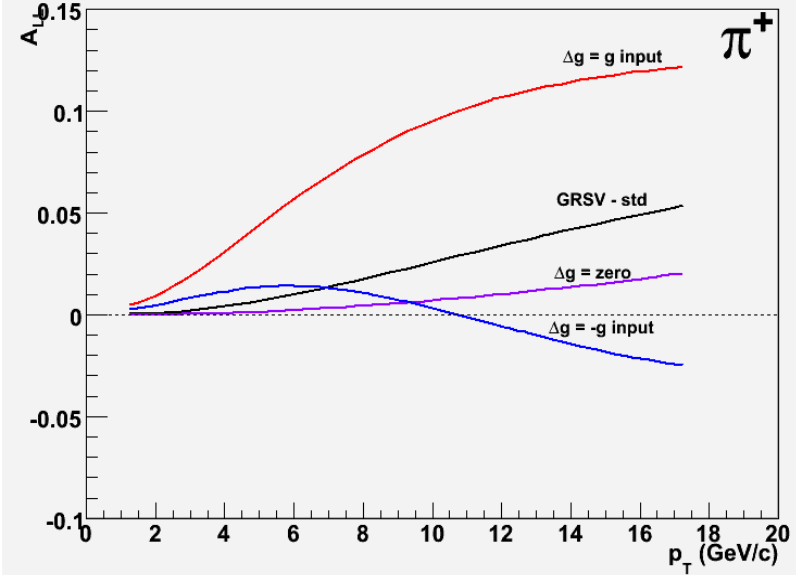
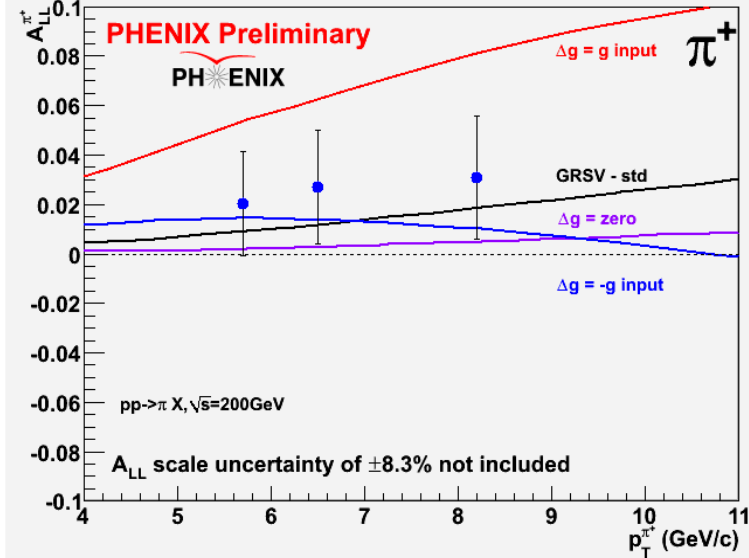
Charged pions above 4.7 GeV identified with RICH.

At higher p_T , qg interactions become dominant: $\Delta q \Delta g$ term.

A_{LL} becomes significant allowing access to the sign of ΔG



Information from $\pi^{+,-}$ Asymmetries

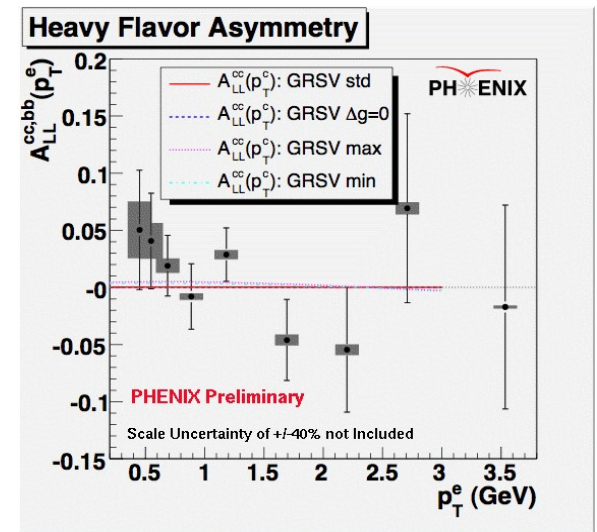
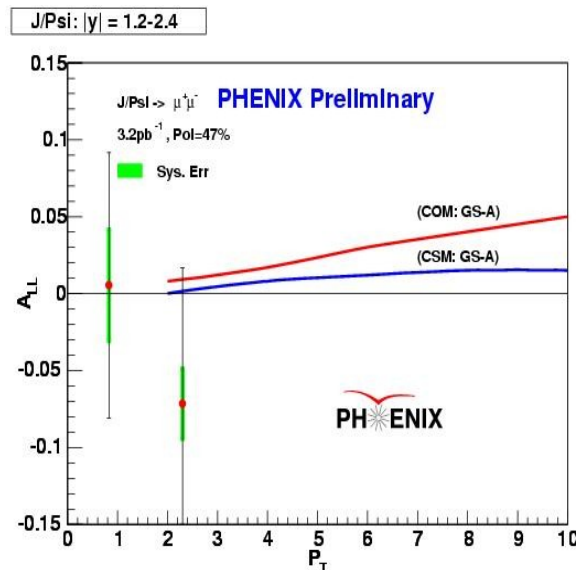
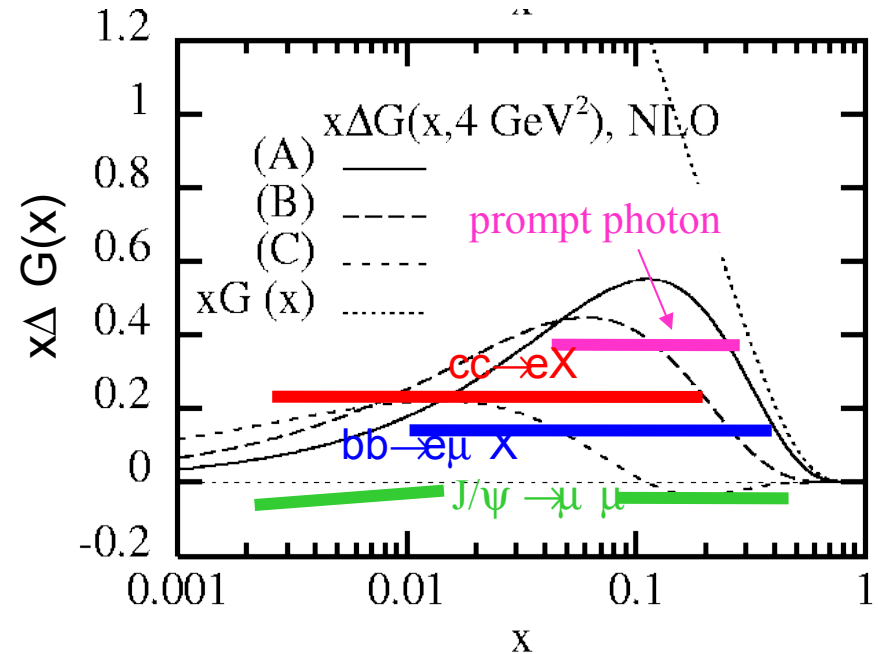
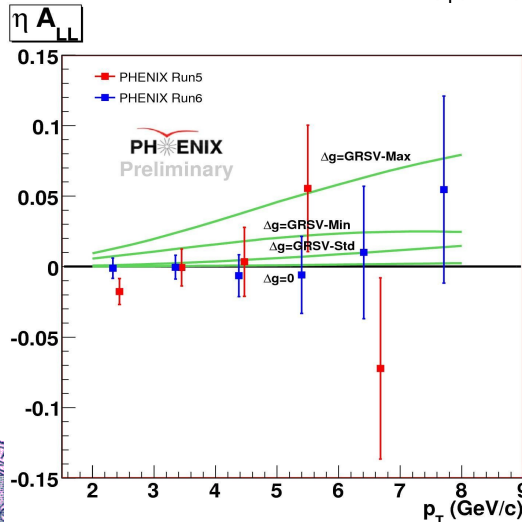
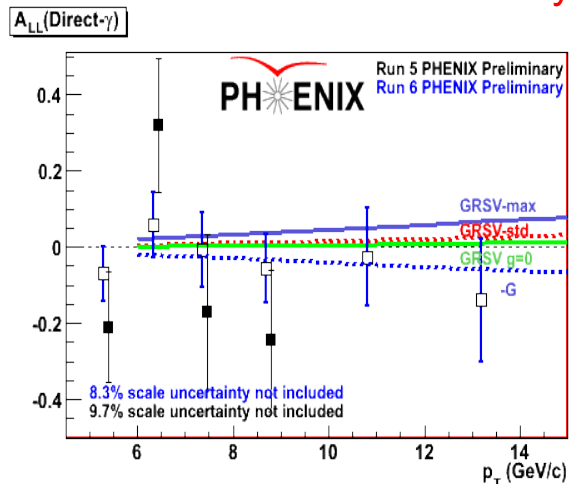


- ❖ Inclusive $\pi^{+,-,0}$ A_{LL} has access to sign $\Delta g(x)$ directly
- ❖ “Model independent” conclusion possible once enough data is available.



❖ Provide access different x range

- Thresholds
- $J/\psi \rightarrow \mu^+ \mu^-$ η range (forward arms)
- Prompt γ : no fragmentation $z=1$
- could help disentangle the contributions from the different quarks and the gluons.
- Rare channels with large background
- **Need more luminosity**



Global Analysis of Polarized PDF's.

- ❖ Results from various channels combined into single results for $\Delta G(x)$
- ❖ Correlations with other PDFs for each channel properly accounted
- ❖ Every single channel result is usually smeared over $x \Rightarrow$ global analysis can do deconvolution (map of ΔG vs x) based on various channel results
- ❖ NLO pQCD framework can be used
- ❖ Global analysis framework already exist for pol. DIS data and being developed to include RHIC pp data, by different groups

One of the attempts of global analysis by
AAC Collaboration using PHENIX π^0
-Preliminary data

Now Run5-Final* and Run6- π^0 data are
available. *Preliminary has now been used in
DSSV Global Fits



Δ G(x) Global Analysis Latest Results

RHIC data set significantly
constraints on the gluon helicity
distribution

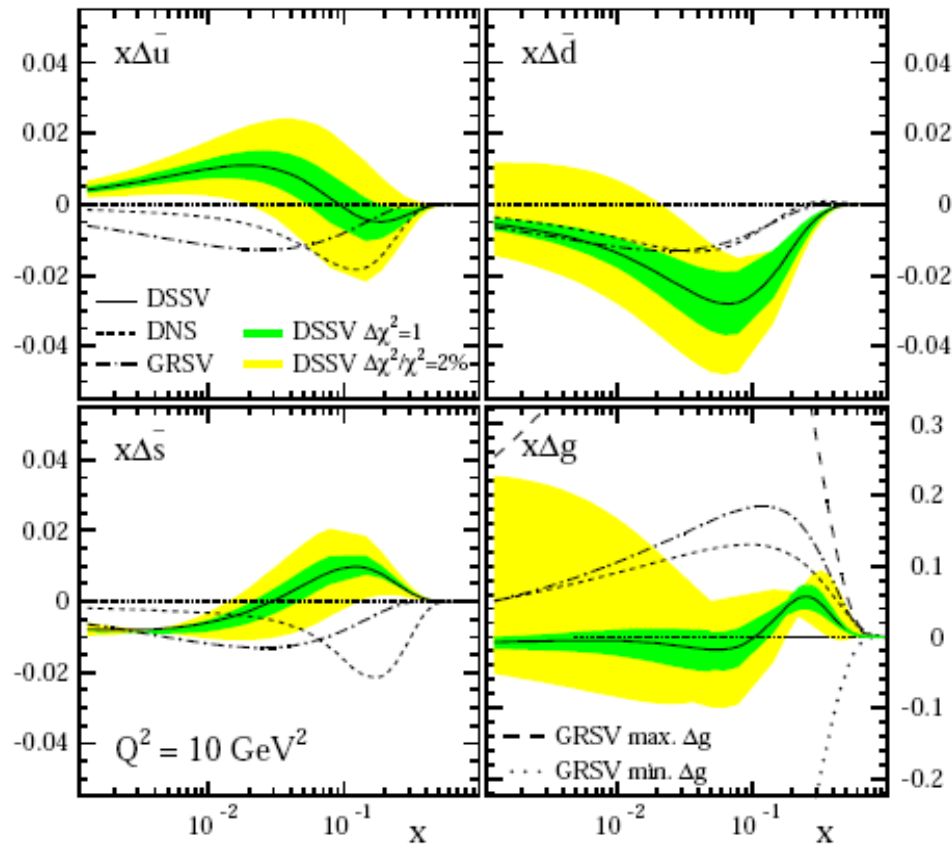


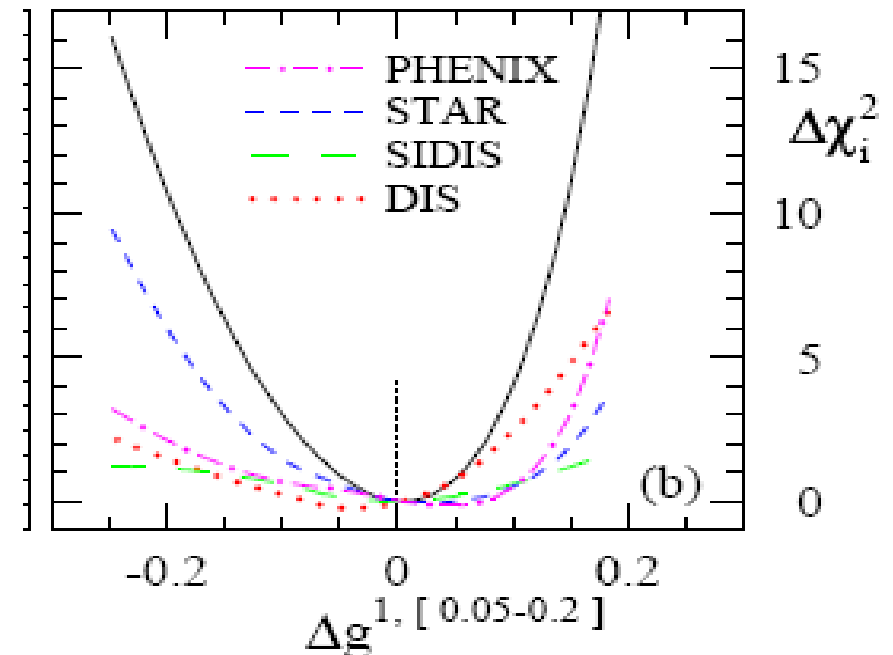
FIG. 2: Our polarized sea and gluon densities compared to previous fits [6, 8]. The shaded bands correspond to alternative fits with $\Delta\chi^2 = 1$ and $\Delta\chi^2/\chi^2 = 2\%$ (see text).

A first demonstration that p-p data can be included in a consistent way in a NLO pQCD calculation.

–Inclusion of theoretical uncertainties and the treatment of experimental ones should and will be improved–

-Flavor dependence of the sea. SU3 symmetry breaking. ??

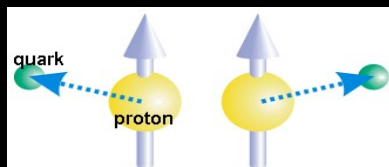
Machine Development for this program is going on as we sit here!!!



Transverse Spin: Origin of the A_N Single Spin Asymmetries.?

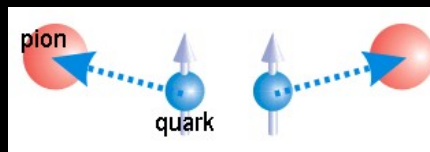
(Sivers effect)

Transversely asymmetric k_t quark distributions



(Collins effect)

spin-dependent fragmentation functions



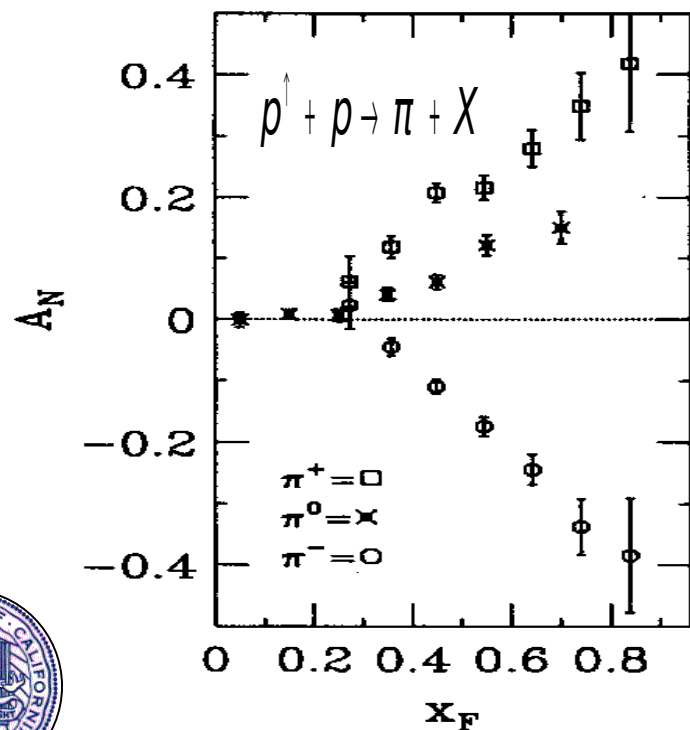
(Twist-3)

quark gluon field interference

$$\delta q, f_{1T}^{\perp q}, L$$

$\sqrt{s}=19.4$ GeV,
 $p_T=0.5-2.0$ GeV/c

Huge A_N measured at E704-FNAL!!!



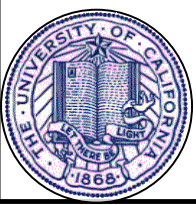
π^0 – E704, PLB261 (1991) 201.

π^{\pm} – E704, PLB264 (1991) 462

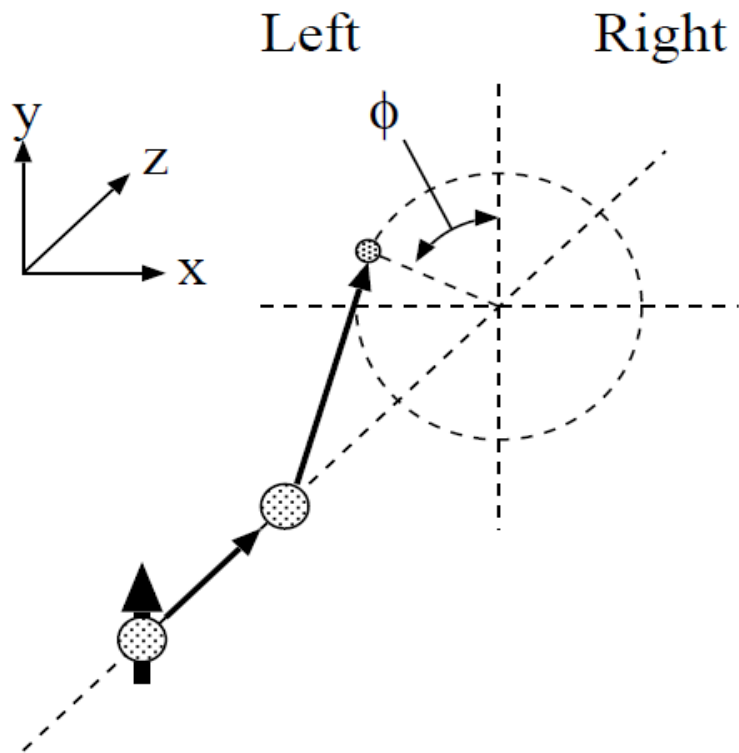
Increase linearly with Feynman x (x_F).

- Extremely bigger than expectation!
- What is the p_T dependence?
- Measure at RHIC? $\sqrt{s} = 62, 200$ GeV.

$$x_F = \frac{p_{z,\pi}}{p_{z,1}} \approx \frac{2E_\pi}{\sqrt{s}}$$



How to measure A_N ?



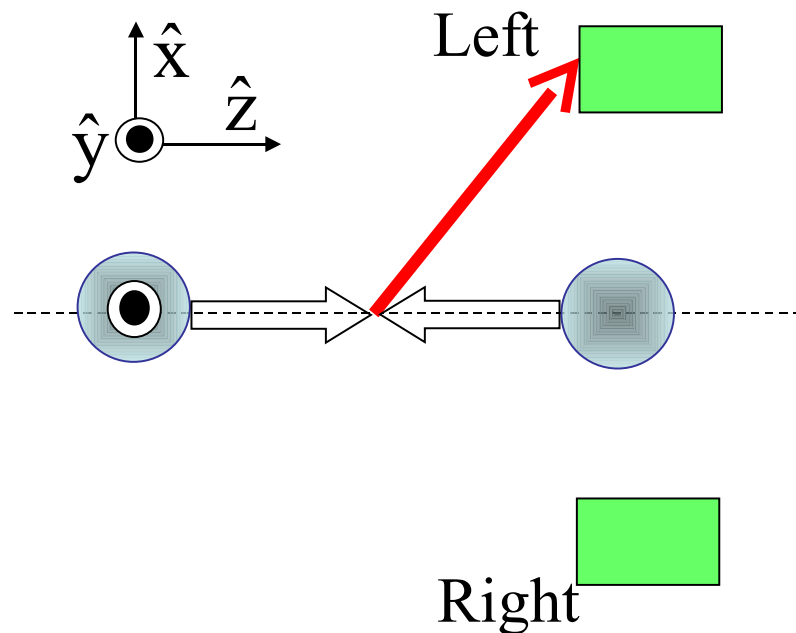
Azimuthal asymmetry is measured by **Double** arms detector (Left-Right)

$$A_N = \frac{\sqrt{N_{\uparrow}^L N_{\downarrow}^R} - \sqrt{N_{\uparrow}^R N_{\downarrow}^L}}{\text{pol.} \left(\sqrt{N_{\uparrow}^L N_{\downarrow}^R} + \sqrt{N_{\uparrow}^R N_{\downarrow}^L} \right)}$$

Square-root-formula

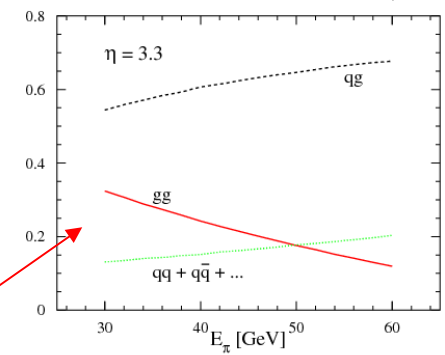
Top view

- Normalization by beam polarization is crucial.**

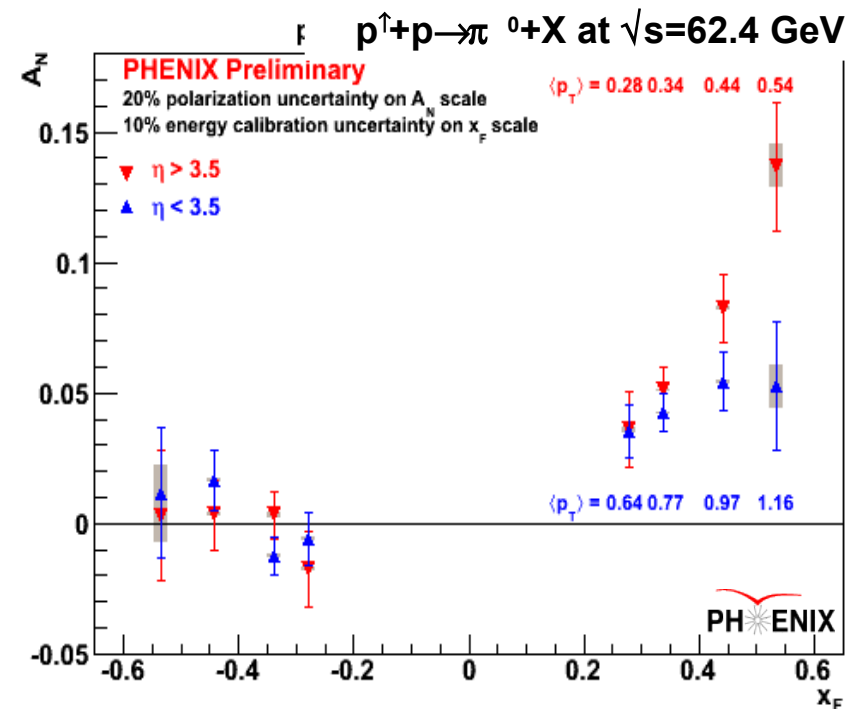
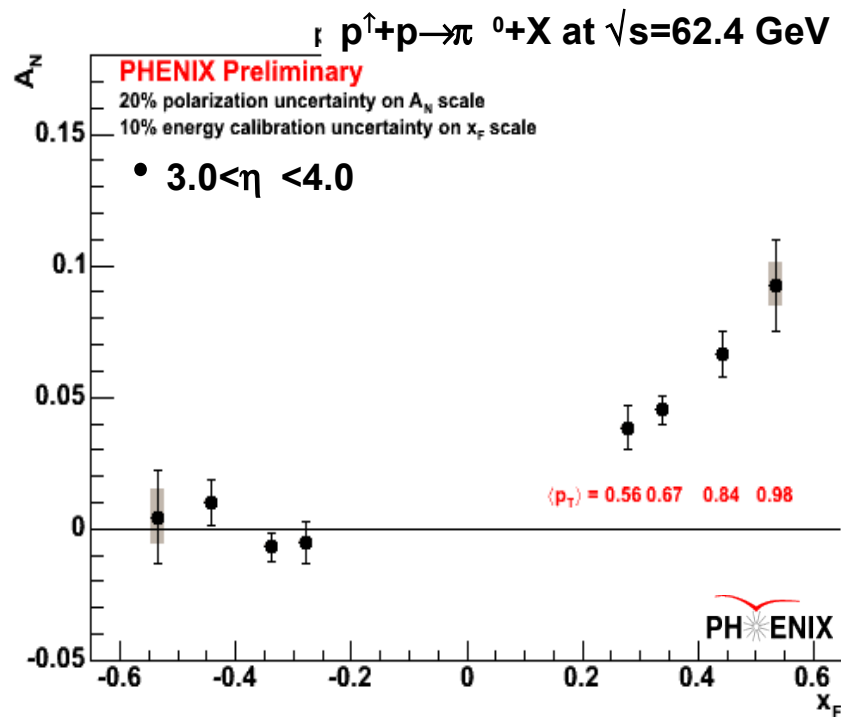


Transverse Spin

π^0 A_N at large x_F



process contribution to π^0 , $\eta = 3.3$, $\sqrt{s} = 200$ GeV



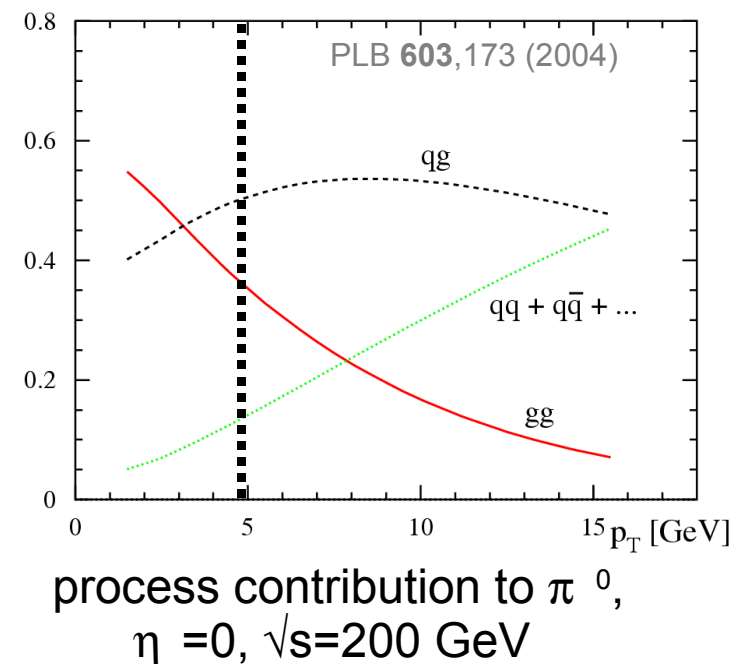
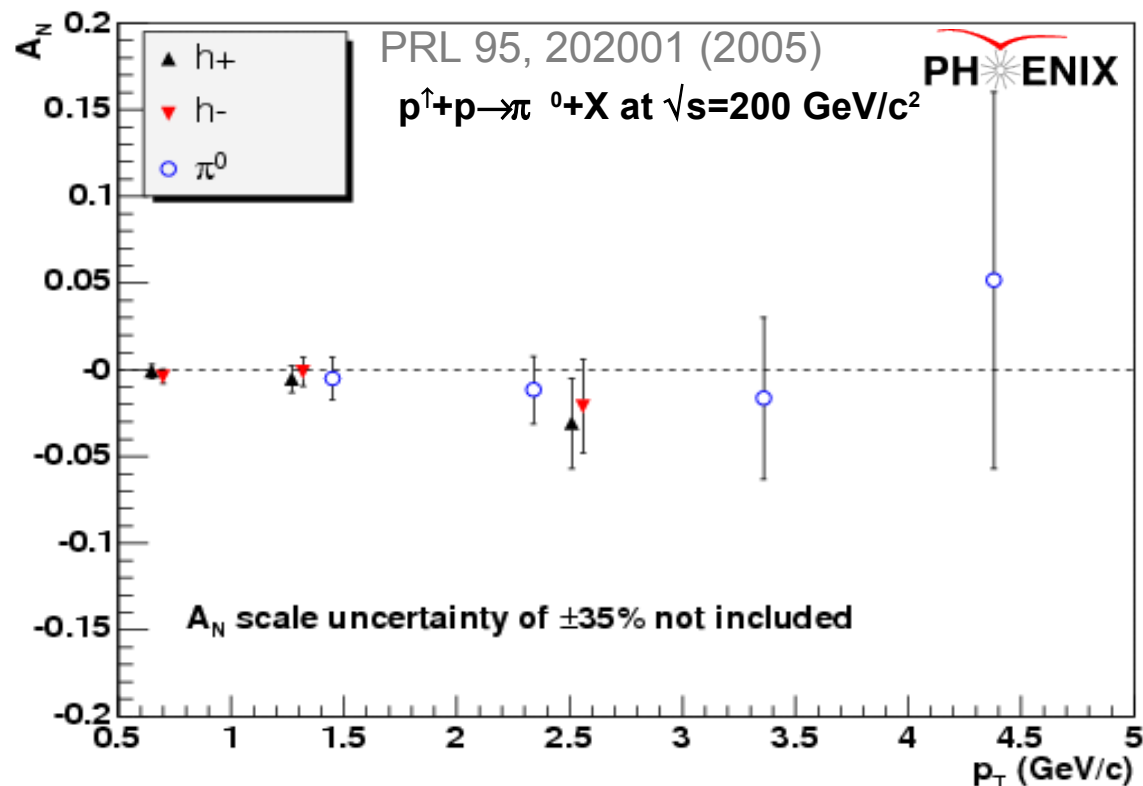
Asymmetry seen in positive x_F (direction of polarized beam), but not in negative x_F .

Large asymmetries at forward $x_F \rightarrow$ Valence quark effect?

x_F , p_T , \sqrt{s} , and η dependence provide quantitative tests for theories

Transverse Spin

Mid-rapidity A_N of π^0 and h^\pm for $y \sim 0$ at $\sqrt{s}=200\text{GeV}$

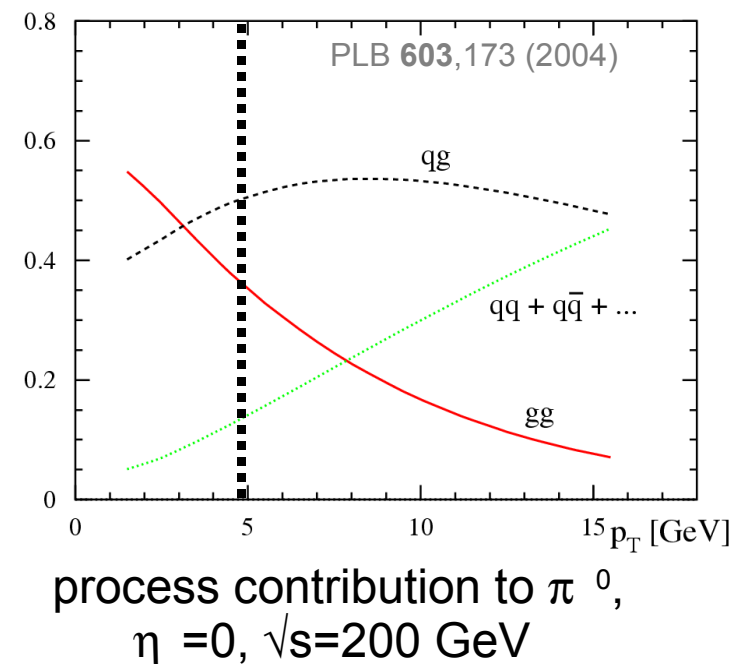
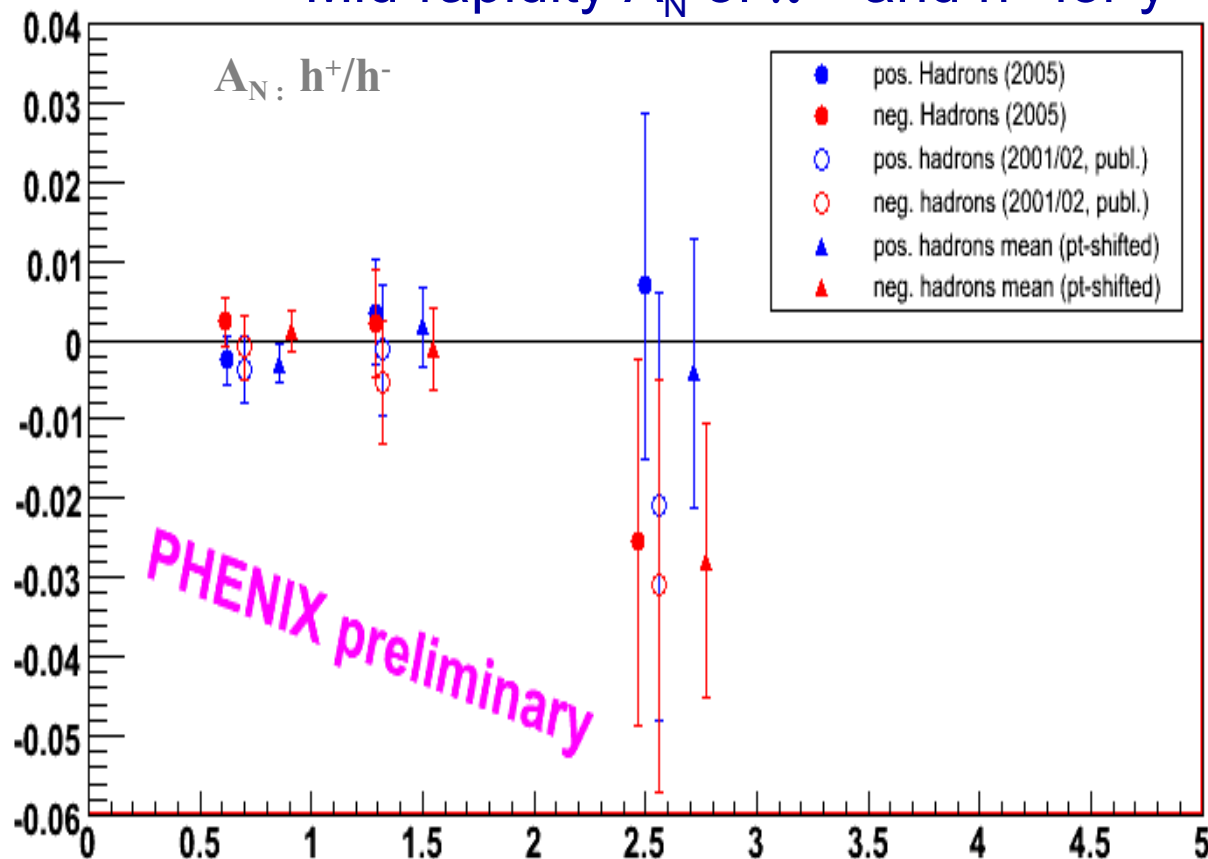


❖ A_N is 0 within 1% \rightarrow interesting contrast with forward π

- Mid-rapidity data at small p_T sensitive to gluons, constrains magnitude of gluon Sivers function (Anselmino et al., PRD 74, 2006)
- What happens if qq sets in (valence quarks) at high p_T ?

Transverse Spin

Mid-rapidity A_N of π^0 and h^\pm for $y \sim 0$ at $\sqrt{s}=200\text{GeV}$



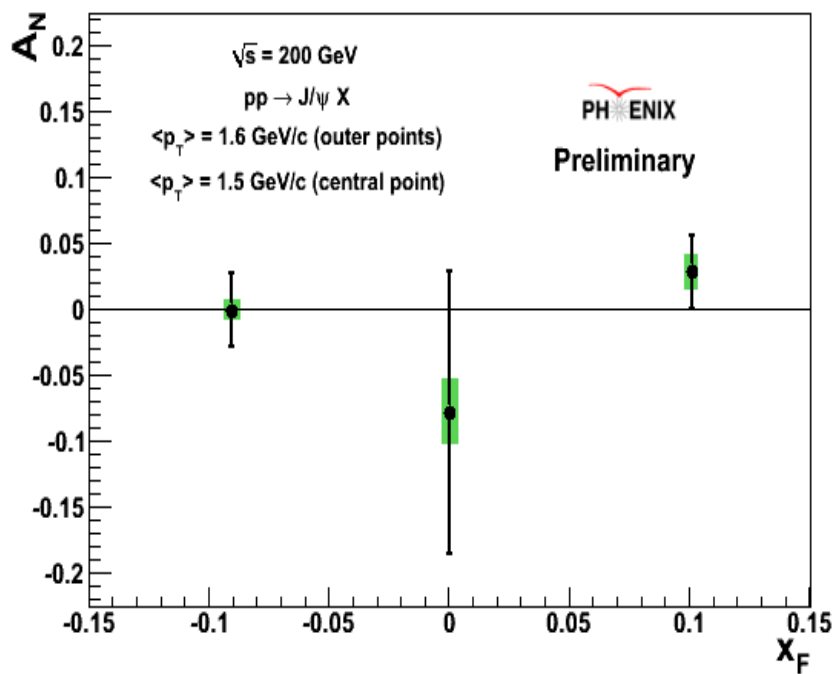
❖ A_N is 0 within 1% \rightarrow interesting contrast with forward π

- Mid-rapidity data at small p_T sensitive to gluons, constrains magnitude of gluon Sivers function (Anselmino et al., PRD 74, 2006)
- What happens if qq sets in (valence quarks) at high p_T ?



Transverse Spin

A_N of J/ψ at $\sqrt{s}=200\text{GeV}$

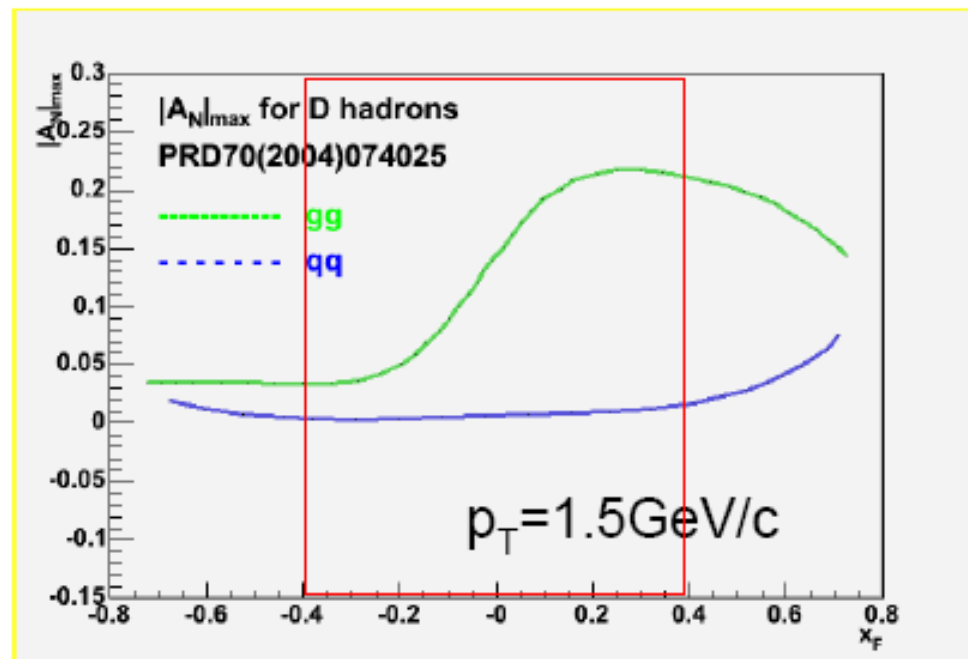


- ❖ May provide insight to J/Psi production mechanism,
- ❖ Sensitive to gluon Sivers as produced through g-g fusion
- ❖ Charm theory prediction is available
 - How does J/ψ production affect prediction?

Theoretical prediction:

For **open charm** production

- quark Sivers function set to its maximum
- gluon Sivers function set to 0
- gluon Sivers function set to its maximum
- quark Sivers function set to 0



Phys. Rev. D 78, 014024 (2008)

Astrid Morreale, Rencontres de Moriond



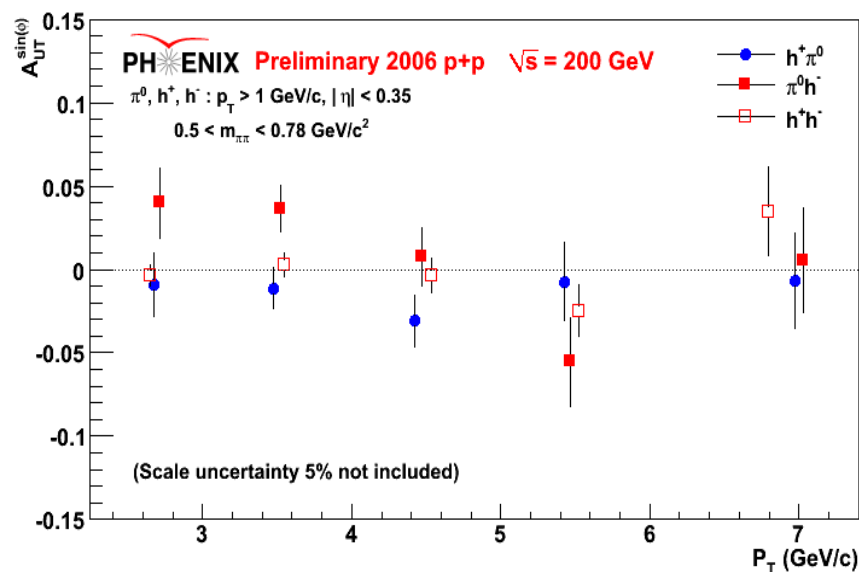
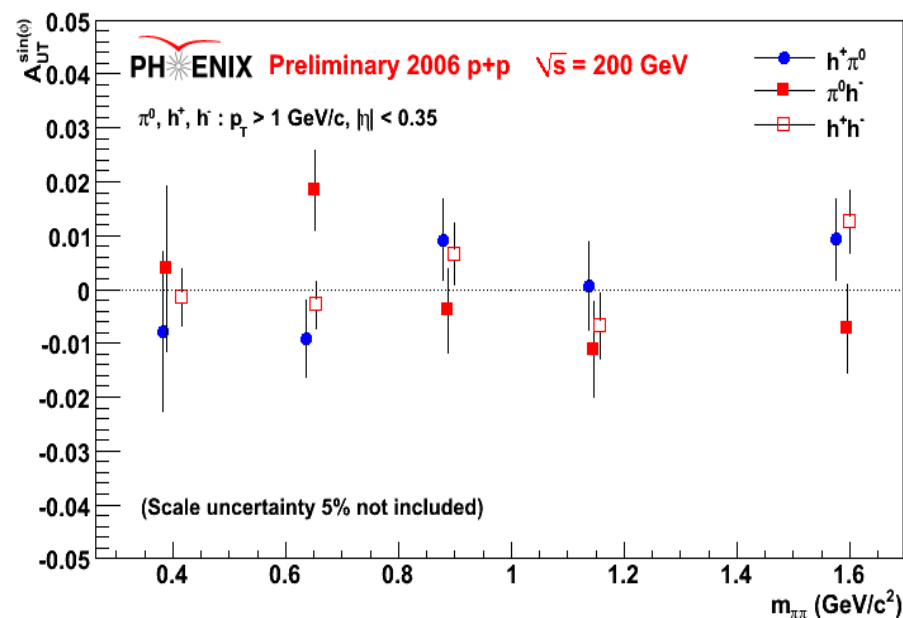
First Look at Transversity with IFF (Interference Fragmentation Function)

Phys. Rev. Lett. 80, 1166 - 1169 (1998), Nuclear Phys. B 420 (1994)565

- Find all hadron pairs
(h^+/π^0), (h^-/π^0), (h^+/h^-)
on the same side of the detector
- Assume all hadrons have a pion mass
 π^0 : $p_T > 1 \text{ GeV}/c$, h : $1 < p_T < 4.7 \text{ GeV}/c$

- Calculate the asymmetry and the analyzing power:

$$A_{UT}(\phi) = \frac{1}{P} \frac{N_{hh}^{\uparrow}(\phi) - RN_{hh}^{\downarrow}(\phi)}{N_{hh}^{\uparrow}(\phi) + RN_{hh}^{\downarrow}(\phi)} = A_{UT}^{\sin\phi} \sin\phi$$



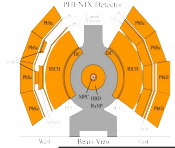
Summary

- ❖ PHENIX is well suited to the study of spin physics with a wide variety of probes.
 - Inclusive π^0 data for A_{LL} has reached statistical significance to constrain ΔG in a limited x-range (~ 0.02 - 0.3).
- ❖ **Need** more statistics (RHIC running time) to explore different (rare) channels for
 - Different gluon kinematics
 - Different mixtures of subprocesses
- ❖ Global Analysis of many channels together with DIS, SIDIS data will give us a more accurate picture of $\Delta g(x)$
- ❖ Upcoming **W program** will give more information about anti-quarks, quarks.
- ❖ PHENIX has an upgrade program that will give us the triggers and vertex information that we need for precise future measurements of ΔG , Δq and new physics at higher luminosity and energy



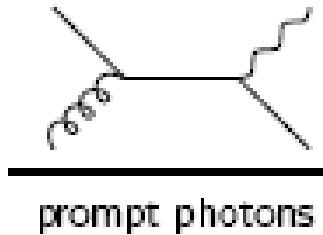
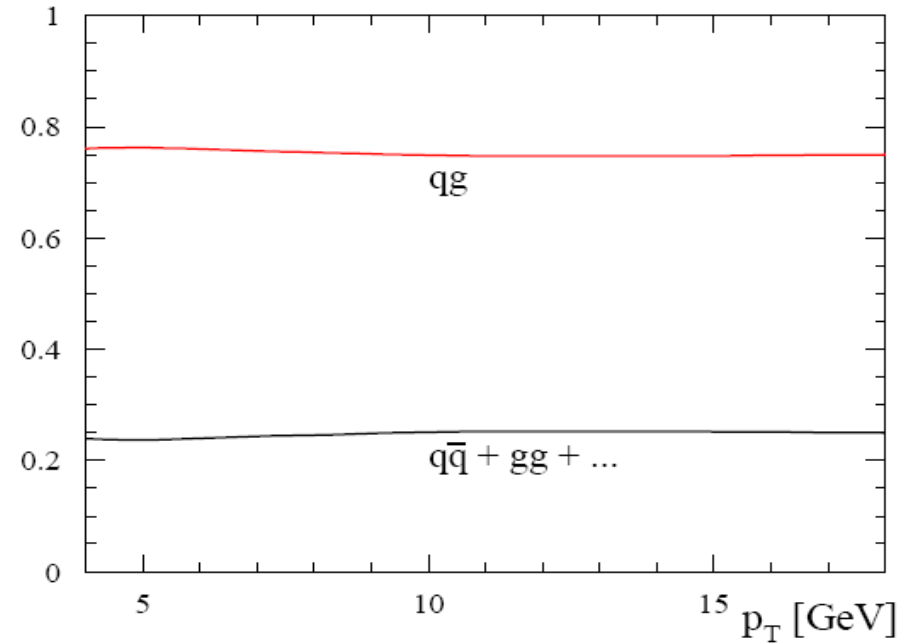
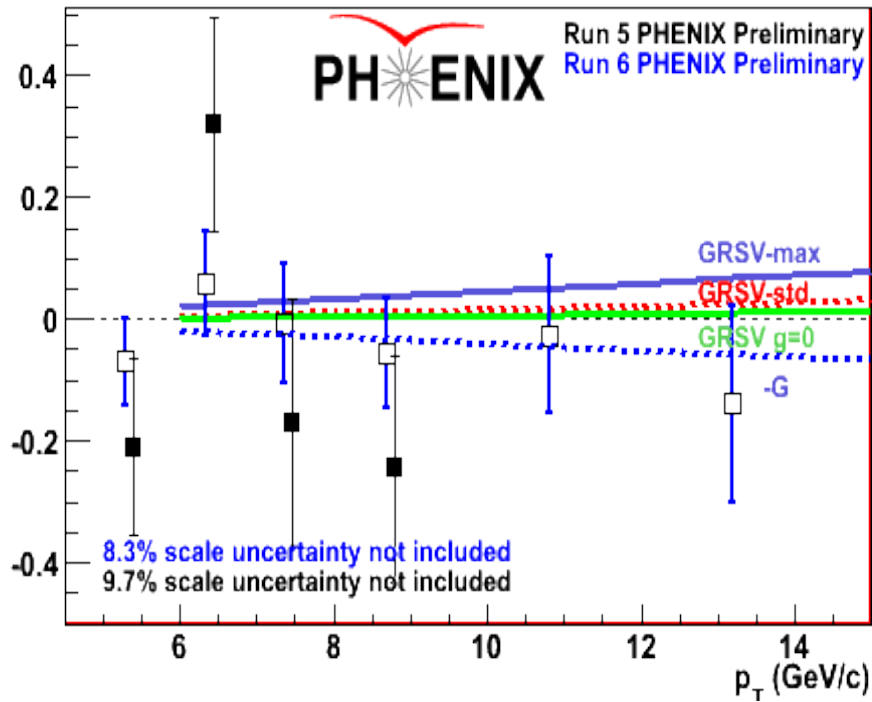
Thank You for Listening





γ Asymmetries: The **Golden Channel**

$A_{LL}(\text{Direct-}\gamma)$



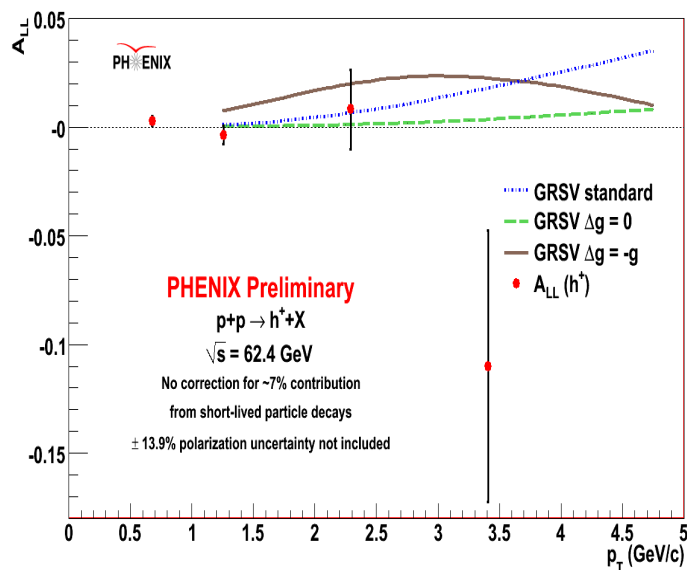
Dominated by qg Compton:

- Small uncertainty from FFs
- Better access to sign of ΔG ($\Delta q \Delta G$)
- Clean "Golden Channel".

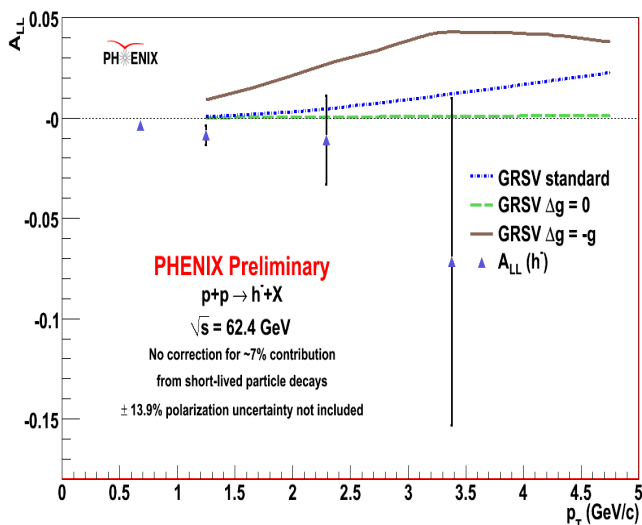
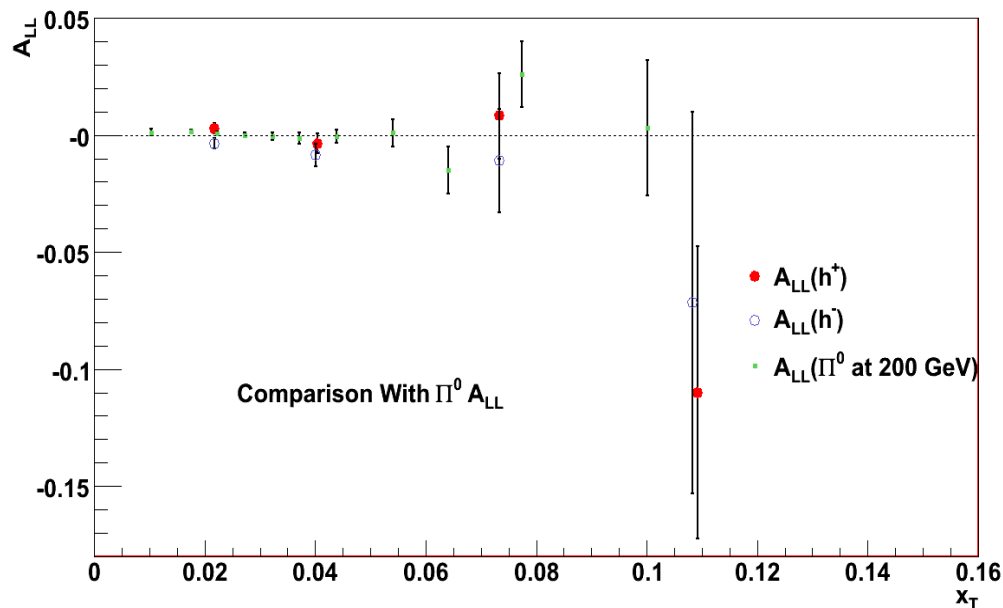
-Rare Probe:Luminosity Hungry



Accessing Different Energies with Charged Hadrons

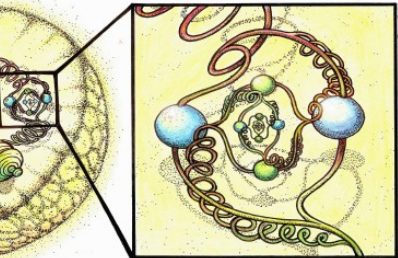


$$x_T = \frac{2p_T}{\sqrt{s}}$$



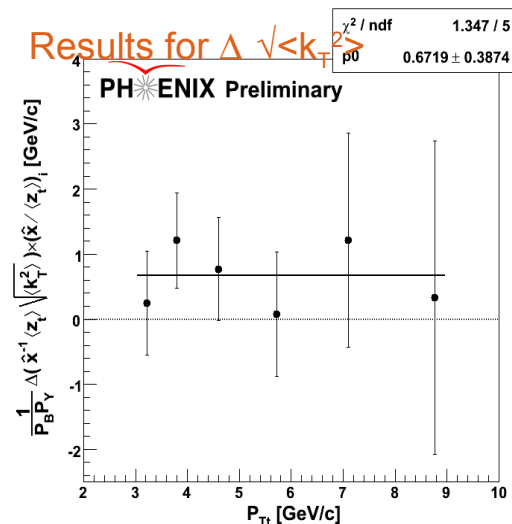
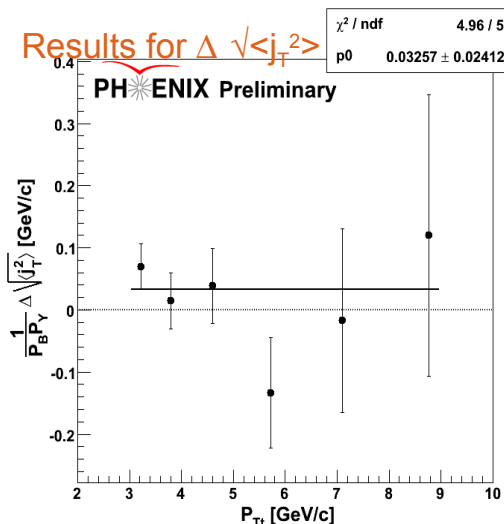
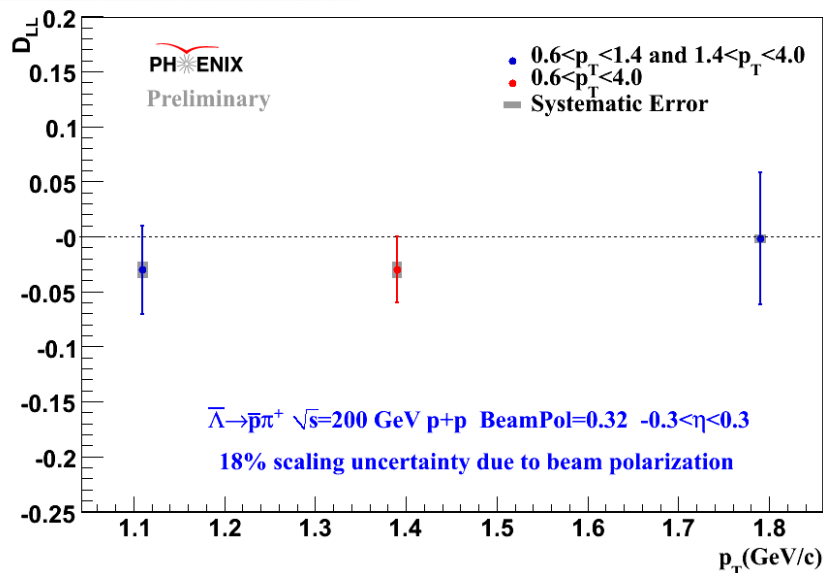
- Comparison with x_T scaling of $\pi^0 A_{LL}$.
- Consistency of asymmetries with results at different center of mass energy.





Transverse Spin

Other asymmetries at $\sqrt{s}=200\text{GeV}$



❖ Strange quark Components via Spin Transfer

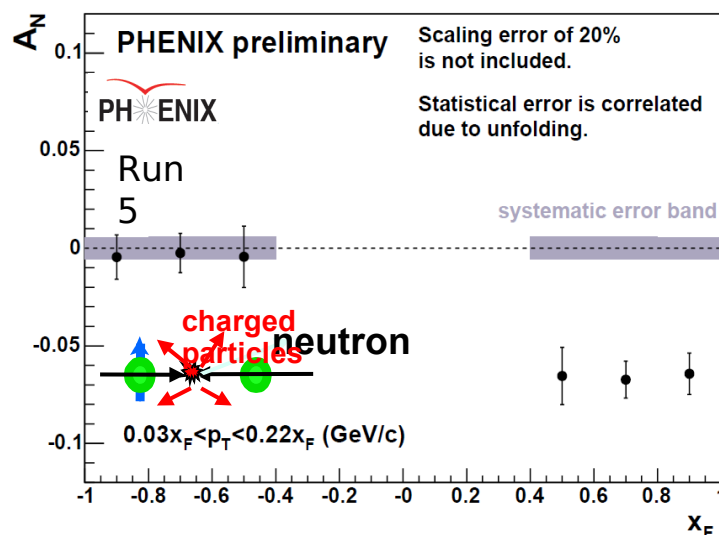
❖ In PHENIX the Self-analyzing decay channel (anti- Λ) has been found to be sensitive to the polarization of the anti-strange sea of the nucleon (See: hep-ph/0511061)

❖ Probing Orbital angular Momentum with k_T

Asymmetries (See: Phys. Rev. D **74**, 072002 (2006))

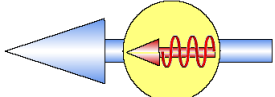
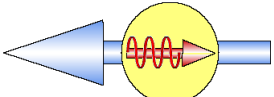
❖ Neutron asymmetries. (See: AIP Conf.Proc.915:689-692,2007)

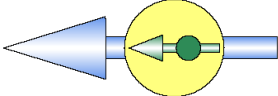
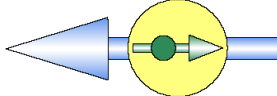
Neutron asymmetry x_F distribution with neutron trigger & MinBias



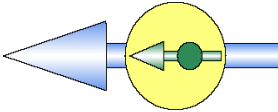
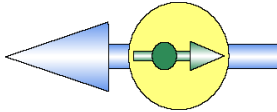
SPIN Dependant Parton Density Functions

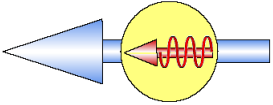
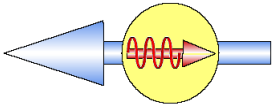
In a proton with positive helicity we can find a parton:

$$g(x, Q^2) = \text{Diagram 1} + \text{Diagram 2}$$



$$q(x, Q^2) = \text{Diagram 3} + \text{Diagram 4}$$



• We then Define Δg , Δq , (Δf) as the probability of finding a quark, gluon or antiquark with spin parallel or anti parallel to the spin of the nucleon.

$$\Delta q(x, Q^2) = \text{Diagram 5} - \text{Diagram 6}$$



$$\Delta g(x, Q^2) = \text{Diagram 7} - \text{Diagram 8}$$



These integrals of Δf multiplied by the spin of the parton f will give the amount of spin carried by each parton*.

*i.e for gluons : **Amount of carried spin $\sim \Delta g * 1$**



Sivers effect and/or Collins- Heppelmann effect?

Theoretical approaches to explain huge SSAs:

- ❑ **Sivers** effect (k_q^\perp is connected to quark orbital angular momentum).
- ❑ **Collins** effect (Analyzer of transversity δq).
- ❑ **Twist3** effect which is related to both initial and final states. Relation of Twist3 to Sivers effect is introduced.

Available Probes at RHIC		
1	$p^\uparrow + p \rightarrow h + X$	Both mix
2	$p^\uparrow + p \rightarrow \text{di-jet} + X$	Sivers?
3	$p^\uparrow + p \rightarrow h + h + X$ (far side)	Separate?
4	$p^\uparrow + p \rightarrow h + h + X$ (near side) $p^\uparrow + p \rightarrow \text{jet} + X$	Collins Sivers
5	$p^\uparrow + p \rightarrow \text{direct } \gamma + X$	Sivers
6	$p^\uparrow + p \rightarrow l^+ l^-$ (Drell-Yan)	Sivers

Relevance of **Twist3** and **Sivers effect** is studied.

PRL97, 082002 (2006)

PRD73, 094017 (2006)

LUMINOSITY

$$L = \rho v$$

$$\frac{dN}{dt} = L\sigma$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{L} \frac{d^2 N}{d\Omega dt}$$

Where

L is the Luminosity.

N is the number of interactions.

ρ is the number density of a particle beam, e.g. within a bunch.

σ is the total cross section.

$d\Omega$ is the differential solid angle.

$\frac{d\sigma}{d\Omega}$ is the differential cross section

For an intersecting storage ring collider:
$$L = f n \frac{N_1 N_2}{A}$$

f is the revolution frequency

n is the number of bunches in one beam in the storage ring.

N_i is the number of particles in each beam

A is the cross section of the beam.